

# **Internet Peer-to-Peer Technology Pricing Schemes as Seen Through the Lens of Principal Agent Theory, Using Napster as an Example Application**

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## ***Introduction***

Of the many services provided on the Internet, a newly emerging service seems to be peer-to-peer file sharing. Peer-to-peer technology enables users to connect to the personal computer of others. One particular application is the providing of an index of MP3 files, making searching for files and connecting to other people's PCs easier.

Some issues being raised of such services are security threats, privacy concerns, and copyright issues. The threats to security and privacy come from the potential to access restricted parts of the user's hard drive, and gain access to private and confidential data. Also, there is much uncertainty about how pricing and cost recovery can be structured for such systems.

In this paper, we suggest the use of incentives using a principal agent framework to create a payment scheme for Internet-related peer-to-peer services. We suggest that the

use of incentives in the payment scheme may provide users with better service and reduce some of the concerns.

### ***Principal Agent Model***

Principal agent theory helps to identify the conditions under which the interests of a party, the principal, will be realized through establishment of a relationship with another party, the agent (Ross 1973, Bowie and Freeman 1992). Principal agent relationships exist when the principal wants a task done, service provided, or output produced and for a variety of reasons, relies on someone else (the agent) to complete the task, provide the service or produce the output (Holstrom 1979). A principal agent problem arises when agents have hidden information about the outcome, are risk averse, or it is not easy for the principal to monitor the agent. Principal agent theory seeks to understand real life problems of loss of control, information asymmetry, monitoring costs, and conflicting goals in organizations. Essentially, principal agent theory suggests that both the principal and the agent exhibit self-interest behaviors.

To enhance sharing of goals with the agent, the principal can enforce certain conditions through a contract, or incentive scheme. The principal's goal is to devise an incentive scheme under which the agent will not shirk. At a minimum, the incentive scheme should satisfy two conditions. First, the agent must be willing to accept the incentive scheme. In other words, the incentive scheme must offer the agent at least as much utility (satisfaction) as the agent's next best alternative, or threshold wage, also known as the agent's reservation utility. This reservation utility is the compensation that the agent could receive by performing some other task rather than work for the principal.

If the agent is not offered at least this amount, the agent will prefer to work for someone else. Second, the scheme should induce the agent to provide the level of effort that the principal desires.

The principal agent problem can be stated mathematically. The mathematical analysis of the principal agent problem in the single-agent case can be stated as follows: The principal (P) wants to hire an agent (A) to perform a service. The output (x) depends on the agent's level of effort (e), and a random factor that the agent does not control. In the context of Internet peer-to-peer technology the random factor may cover a number of things such as a temporary shutdown of some Internet servers, connection problems due to heavy bandwidth, peers intentionally logging off and thus severing the connection, etc. The principal's goal is to devise an incentive scheme (I) that maximizes his utility ( $U_P$ ) while ensuring that the agent receives a utility ( $U_A$ ) that is at least as much as the agent's reservation utility (threshold wage) K. Thus, following Holstrom (1979) the principal's problem becomes:

$$\max E\{U_P(x-I(x))\} \tag{1}$$

$$\text{subject to } E\{U_A(I(x))\} \geq K \tag{2}$$

$$\text{subject to } E\{U_A(I(x), e)\} \geq E\{U_A(I(x), e')\} \text{ for all } e' \neq e \tag{3}$$

This says that (1) the principal wants to choose an incentive scheme  $I(x)$  to maximize her expected utility, subject to (2) the agent's utility must be at least as much as his reservation utility (or threshold wage), and to (3) effort level e is more desirable (results in at least as much payment) to the agent than any other effort level.

## ***Application to Napster***

One of the most popular peer-to-peer services now available is Napster. Napster (<http://www.napster.com>) acts like a music search engine, maintaining servers that contain directories of music files kept by registered users. The music files themselves remain on the user's hard drive. Napster's software provides users with a list of active users that possess the files they are looking for, provides a connection between the cooperating users' computers, and transfers the desired file. This service is currently free and has not paid copyrighted-related royalties. The popularity of this and similar services has alarmed many elements of the music industry and has led to them suing the company for violating copyright laws ([http://www.riaa.com/Napster\\_legal.cfm](http://www.riaa.com/Napster_legal.cfm)). In addition to these intellectual property issues, the security of computers registered with Napster and the privacy of information in them is also of concern.

Some possible principal agent relationships in this area are between the Internet user and Napster, recording artists and Napster, and the music industry and Napster. These are multiple principal single agent relationships. In the first relationship, users (acting as principals) desire to find and download MP3 music files. Napster (the agent) provides a service by maintaining an index of MP3 databases. If recording artists or others in the music industry (principals) decided to use Napster-like services as a medium to sell their music or other goods or services, Napster could act as an agent providing marketing services as well.

Napster currently receives no fee for its services. Potential investors have suggested that it collect subscription fees from members, persuade record labels to use the service as a marketing tool, or act as an e-commerce outlet for CDs (Harmon 2000;

Kover 2000). But incentive-based pricing schemes tied closely to the agent's effort (where the effort includes that required to maintain security and privacy, and protect copyrights) may provide an alternative to subscription-based plans, and are more likely to ensure that the agent will provide an effort level required to maintain the principals' desired level of service.

In the principal agent paradigm, the principal's goal is to provide an incentive scheme under which the agent will not shirk. In other words, the users want Napster to maintain an updated index properly (without broken links, flaky or malicious computers, etc.) Napster or another service could maintain a list of only those files that have a copyright protection mechanism (which have proven in the past to be wildly unpopular) or could exert more efforts to preserving the security and privacy of the principals' computer systems.

If we assume that Napster, the agent, is (somewhat) risk-averse (i.e. prefers a "sure thing" to a bet of equal value), the principals can offer an incentive scheme with a risk premium that provides enough incentives so that the agent does not shirk and accepts a certain degree of risk.

For the various relationships, the output and effort level can be defined as follows:

When the principal is the Internet user, the agent's output  $x$  can be the money that the user saves by downloading music files from Napster rather than buying CDs or tapes or other traditional physical embodiments of the music. The agent's level of effort  $e$  can be defined as the effort required for maintaining the index of files (in terms of manpower, equipment, etc) in monetary values.

In other relationships, the recording artist (or record companies in their role as copyright holders) can offer an incentive scheme that pays according to the number of times their music is searched for and downloaded.

As an example, consider the problem facing the Internet user of employing Napster to provide an indexing service. For simplicity, assume that Napster can either exert a high level of effort ( $e = 1$ ) or a low level of effort ( $e = 0$ ). We can define a high effort level ( $e = 1$ ) as effort required to maintain the index properly in addition to providing security, privacy and copyright protection. Low effort level ( $e = 0$ ) can be defined as the minimum effort required to maintain the index. In this example, let the output be  $x = 1$  if the desired file is found and downloaded with traditional security, privacy, and copyright protection, and  $x = 0$  if it is not. In this example, effort level and output are not dollar values, but binary values. Of course, this example can be expanded to include more than two levels of effort and output, making the model more realistic. For example, output can be high, medium or low. High output can be when files are downloaded with traditional copyright, security and privacy protection, medium output can describe situations where files are downloaded without these protections, and low output can be when files cannot be downloaded. To make the model even more realistic, output  $x$  can be a continuous random variable that represents user profit (or savings) in dollar values. These other models may be developed later, but aren't necessary to make the point of this paper, about the utility of principal agent theory for describing certain Internet peer-to-peer transactions. For the scope of this paper, we will limit our effort and output to binary values.

Even with the agent exerting maximum effort, random factors outside his control (as described above) affect the user's ability to download successfully. Thus we assume that the probability of successfully downloading the file, given the agent's effort level, is as shown in Table 1.

Table 1. Outcome Probability

		effort (e)	
		0	1
output (x)	0	<b>0.5</b>	<b>0.25</b>
	1	<b>0.5</b>	<b>0.75</b>

Limiting our discussion to compact discs (CDs) for simplicity, if we assume that one CD costs \$15, and there are approximately 15 songs in one CD, the Internet user stands to save the expense of 1 (dollar) for each file they download rather than buying in the traditional manner a CD containing the desired file. Additionally, the user will stand to make a savings of 0 if the file is not downloaded properly. The Internet user's utility equals the output minus what Napster charges him:  $U_p(x) = x - I(x)$ . If we assume that it costs Napster 20 cents to provide a unit of effort per file ( $0.20e$ ), then Napster receives a certain degree of utility from its net profit, which would be the incentive scheme (what it charges the user) minus the cost of effort for that one file. Napster's utility in this case can be represented as  $\sqrt{I(x)} - 0.20e$ . Here, we are using a special case of a class of functions known as power functions to describe utility. Power functions take the form  $U = BX^\alpha/\alpha$ . In this example,  $B = 0.5$ ,  $X = I(x)$ ,  $\alpha = 0.5$ . As  $\alpha$  decreases, risk aversion increases (Besanko et al. 1996).

Our last assumption is about Napster's reservation utility  $K$ . If we assume that Napster can get the same utility by charging companies to display ads on its Web site, this value can be used as the reservation utility. A typical rate for general, not-very-targeted web sites is 1 cent per person per visit to the Web site. (Wilson 2000). Since Napster is a relatively popular Web site, for this example we will assume that the reservation utility is 0.05 (i.e. Napster can get the same utility by charging 5 cents per person for an ad on its web site, a value 5 times greater than a general Web site).

Table 2 summarizes our assumptions up to this point.

Table 2. Summary of Variables

<b>Variables</b>	<b>Values</b>	<b>Description</b>
Effort Level $e$	High ( $e = 1$ )	Effort to maintain index and provide security, privacy and copyright protection
	Low ( $e = 0$ )	Minimum effort required to maintain index
Output $x$	Successful ( $x = 1$ )	File is downloaded with security, privacy and copyright protection
	Unsuccessful ( $x = 0$ )	File is not downloaded properly
Principal's Savings	1 or 0	Principal saves a dollar for every file downloaded
Cost of Effort	$0.20e$	Each unit of effort costs 20 cents/file
Agent's Utility	$\sqrt{I(x)} - 0.2e$	The agent's utility is the utility received from his proceeds, minus the cost of effort
Reservation Utility $K$	0.05	Napster can receive the same utility by charging 5 cents/person for banner ads

Given all this, we can now formulate the Internet user – Napster relationship as a specific instance of principal agent theory, where the principal’s problem becomes:

$$\max E\{x - I(x)\} \quad (4)$$

$$\text{subject to } E\{\sqrt{I(x)} - 0.2e\} \geq 0.05 \quad (5)$$

$$\text{subject to } E\{\sqrt{I(x)} - 0.2e\} \geq E\{\sqrt{I(x)} - 0.2e'\} \text{ for all } e' \neq e \quad (6)$$

This says that the Internet user wants to (4) maximize her utility which is the money he saves from downloading a file (as opposed to buying) minus the payment to Napster, subject to the constraints that (5) Napster’s utility should be at least as much as the utility it would receive from displaying banner ads (5 cents), and (6) exerting effort level  $e$  will yield at least as much utility to Napster as any other level.

If a high level of effort ( $e = 1$ ) is desired, we can use the probabilities from Table 1 to describe the specific problem:

$$\max \{(1 - I(1)) * 0.75 + (0 - I(0)) * 0.25\} \quad (7)$$

$$\text{subject to } (\sqrt{I(1)} - 0.2) * 0.75 + (\sqrt{I(0)} - 0.2) * 0.25 \geq 0.05 \quad (8)$$

$$\text{subject to } (\sqrt{I(1)} - 0.2) * 0.75 + (\sqrt{I(0)} - 0.2) * 0.25 \geq (\sqrt{I(1)} * 0.5 + (\sqrt{I(0)}) * 0.5) \quad (9)$$

In particular, (7) says that the principal’s expected utility for a high effort level ( $e = 1$ ) should be maximized, subject to the constraints that (8) Napster’s expected utility is at least as much as 5 cents, and (9) exerting a high level of effort ( $e = 1$ ) should provide Napster with at least as much utility as exerting a low level of effort ( $e = 0$ ).

This maximum is reached when  $I(1) = 0.2025$  and  $I(0) = 0.1225$ . Furthermore, the principal’s expected utility, Napster’s expected utility when providing a high effort level

( $e = 1$ ), and Napster's expected utility when providing a low effort level ( $e = 0$ ) are respectively  $E(U_P) = 0.5675$ ,  $E(U_A(e=1)) = 0.5$ , and  $E(U_A(e=0)) = 0.5$ . For a detailed description of how these values were obtained, please refer to Appendix A.

In this example, we will round the numbers so that they can be discussed in terms of dollars and cents. In other words,  $I(1) = 0.20$ ,  $I(0) = 0.12$ , and  $E(U_P) = 0.57$ . Thus, for every file downloaded, the Internet user pays 20 cents to Napster if it is successful and 12 cents if it is not. This gives the user an expected utility (or expected net savings) of 57 cents. Also, the agent's utility for the two effort levels is equal. In other words, the agent is indifferent between the two actions and their associated rewards. In such cases, we can assume that factors outside of the principal agent paradigm (such as reputation and ethical values) will induce the agent to select the action more favored by the principal (Bowie and Freeman 1992).

Other principal agent relationships in Napster applications can be examined similarly. For example,

- Principal is the recording artist: If the recording artist makes a profit of 50 cents for his music every time it is downloaded, the artist's utility becomes  $U_P(x) = 0.5x - I(x)$ . Also, assuming that a principal agent relationship between the Internet user and Napster had been established, Napster could now announce its reservation utility  $K$  as the utility it receives from payment by Internet users. Thus, the new reservation utility can equal  $E\{I(x)\} = I(1)*0.75 + I(0)*0.25 = 0.20*0.75 + 0.12*0.25 = 0.18$ . In other words,  $K$  equals 18 cents.

- Principal is the recording company, acting as copyright holder: If the company also makes a profit of 50 cents for each file that is downloaded, the company's utility becomes  $U_P(x) = 0.5x - I(x)$ . For the same reason stated earlier, the reservation utility  $K$  can be 18 cents for the relationships between recording companies and Napster.

Of course, it may be possible that the models for each relationship have different reservation utilities. For example, when recording companies are the principals, Napster's reservation utility can be the compensation it would receive if it were to provide copyright protection services for other companies. Even within a relationship, Napster could have different reservation utilities for different principals. An example would be if (for recording artists) Napster were to have a lower reservation utility for a startup band than for a famous profitable group. This would give starting artists a better chance to make themselves known and make some money. But usually, in principal agent theory, principals and agents are thought of as rational, utility-maximizing beings (Fukuyama 1995). Therefore, reservation utilities that are not self-maximizing to the agent may not be realistic.

Table 3 shows the resulting incentive schemes when the variables are defined as above.

Table 3. Sample Incentive Schemes Per File  
 (when the principal wants to induce a high level of effort)

	<b>Savings/ Profit (\$)</b>	<b>I(x=1)</b>	<b>I(x=0)</b>	<b>Principal's Expected Net Profit E{x-I(x)}</b>	<b>K (Reservation Utility )</b>
<b>Internet User</b>	1	0.20	0.12	0.57	0.05
<b>Recording Artist</b>	0.5	0.34	0.05	0.11	0.18
<b>Music Company</b>	0.5	0.34	0.05	0.11	0.18

These results illustrate savings that the Internet and in particular peer-to-peer technology can bring to distribution of intellectual property. The three principals named above each improve their positions over today's situation by either receiving compensation for the sales of their intellectual property, or being guaranteed that the merchandise they receive is not a bootlegged copy and does not harm the security of their computers or compromise their privacy. Even at this early stage of analysis, large numbers of players in the supply chain drop out, and thus provide resultant savings to the end-user. In particular there are no costs, and none reflected, for agents, music reviewers, fabrication companies, distribution companies, jacket design firms, etc. To the extent that these and other players provide services valuable to the user, they could themselves be agents (to one or more of the principals above) or their services could be provided by an agent with a broader mission or the same agent at a higher price. For example, if Napster were to provide (say) music review services as well, assume that this would increase their effort costs to 30 cents per (reviewed) file, rather than the 20 cents charged in the previous example. Napster's utility would now equal  $\sqrt{I(x)} - 0.3e$ .

Using this utility, the problem would now be

$$\max E\{x - I(x)\} \quad (10)$$

$$\text{subject to } E\{\sqrt{I(x)} - 0.3e\} \geq 0.05 \quad (11)$$

$$\text{subject to } E\{\sqrt{I(x)} - 0.3e\} \geq E\{\sqrt{I(x)} - 0.3e'\} \text{ for all } e' \neq e \quad (12)$$

When solving for a high level of effort, the maximum would be obtained when  $I(1) = 0.42$ , and  $I(0) = 0.30$ , and the principal's expected utility would be  $E\{x - I(x)\} = 0.36$ .

The Internet user would now pay either  $I(1) = 42$  cents for a successful download, or  $I(0) = 30$  cents for an unsuccessful one with an expected net savings of 36 cents. The result would be that the Internet user would now only have an expected savings of 36 cents not 57 cents (in the case where effort was 20 cents per unit) per file. But he or she would have the benefit of music reviews for that file.

## ***Conclusion***

Internet-related peer-to-peer services may become very widespread, but there are concerns regarding security and copyright issues. Price models for these services have not yet been thoroughly explored though several have been suggested. This paper introduced the use of incentive schemes as a means for these service providers to receive payment for their services while simultaneously reducing the risks related to security, privacy, and copyright issues.

Principal agent theory and the model it uses acts more as a guideline than a solution. It can be used to examine the relationship between risk attitudes, effort levels, and the use of incentives.

## **References**

- Besanko, D., Dranove, D., and Shanley, M. (1996). *Economics of Strategy*, John Wiley, New York.
- Bowie, N. E. and E. R. Freeman, Eds. (1992). *Ethics and Agency Theory*, Oxford, Oxford University Press.
- Harmon, A. (2000). "Powerful Music Software has Industry Worried." New York Times (March 7 2000).
- Holstrom, B. (1979). "Moral Hazard and Observability." Bell Journal of Economics **10**: 74-91.
- Huang, C. J. and P. S. Crooke (1997). *Mathematics and Mathematica for Economists*, Blackwell Publishers.
- Kover, A. (2000). Napster: The Hot Idea Of The Year. Fortune **142** (June 26 2000).
- Ross, S. A. (1973). "The Economic Theory of Agency: The Principal's Problem." The American Economic Review **63**(2): 134-139.
- Wilson, R. L. (2000). Using Banner Ads to Promote Your Website. Web Marketing Today (July 1 2000).

## APPENDIX A. DERIVATION OF THE MODEL

Optimization problems with constraints can be solved using the Lagrange Multiplier method. The Lagrange Multiplier method is used to find the extremum of  $f(\chi_1, \chi_2, \dots, \chi_n)$  subject to the constraint  $g(\chi_1, \chi_2, \dots, \chi_n) \leq C$ , where  $f$  and  $g$  are functions with continuous partial derivatives. For a more detailed discussion of the Lagrangian Multiplier method, see Huang and Crooke (1997).

To find the extremum, each of the partial derivatives of the Lagrangian function should equal 0 where the Lagrangian function  $L$  is defined as

$$L = f(\chi_1, \chi_2, \dots, \chi_n) + \lambda g(\chi_1, \chi_2, \dots, \chi_n)$$

where  $\lambda$  is the Lagrangian multiplier

In other words,

$$\partial L / \partial \chi_i = 0 \text{ (for all } i = 1 \dots n \text{)}$$

$$\partial L / \partial \lambda = 0$$

The stationary point ( $\chi_1 = x_1, \chi_2 = x_2, \dots, \chi_n = x_n$ ) that satisfies the above conditions is the point where  $f(\chi_1, \chi_2, \dots, \chi_n)$  is an extremum under the given constraint.

This method can be extended to more than one constraint by adding additional Lagrangian multipliers as needed.

In our principal agent problem involving Napster, the principal's problem was generally stated as:

$$\max E\{x - I(x)\} \tag{4}$$

$$\text{subject to } E\{\sqrt{I(x)} - 0.2e\} \geq 0.05 \tag{5}$$

$$\text{subject to } E\{\sqrt{I(x)} - 0.2e\} \geq E\{\sqrt{I(x)} - 0.2e'\} \text{ for all } e' \neq e \tag{6}$$

To induce a high level ( $e=1$ ) of effort, the problem is:

$$\max \{(1 - I(1)) * 0.75 + (0 - I(0)) * 0.25\} \quad (7)$$

$$\text{subject to } (\sqrt{I(1)} - 0.2) * 0.75 + (\sqrt{I(0)} - 0.2) * 0.25 \geq 0.05 \quad (8)$$

$$\begin{aligned} \text{subject to } (\sqrt{I(1)} - 0.2) * 0.75 + (\sqrt{I(0)} - 0.2) * 0.25 \\ \geq (\sqrt{I(1)}) * 0.5 + (\sqrt{I(0)}) * 0.5 \end{aligned} \quad (9)$$

These can be further simplified as follows:

$$\max (0.75(1 - I(1)) - 0.25I(0)) \quad (7a)$$

$$\text{subject to } 0.75\sqrt{I(1)} + 0.25\sqrt{I(0)} - 0.25 \geq 0 \quad (8a)$$

$$\text{subject to } 0.25\sqrt{I(1)} - 0.25\sqrt{I(0)} - 0.2 \geq 0 \quad (9a)$$

Formulas (7a through (9a) can be seen as an optimization problem with two variables  $I(1)$  and  $I(0)$ . For readability, let us substitute  $\chi_1$  for  $I(1)$  and  $\chi_2$  for  $I(0)$ . Using the Lagrangian Multiplier method, (7a) through (9a) becomes:

$$\max f(\chi_1, \chi_2) = 0.75(1 - \chi_1) - 0.25\chi_2 \quad (7b)$$

$$\text{subject to } g_1(\chi_1, \chi_2) = 0.75\sqrt{\chi_1} + 0.25\sqrt{\chi_2} - 0.25 \geq 0 \quad (8b)$$

$$\text{subject to } g_2(\chi_1, \chi_2) = 0.25\sqrt{\chi_1} - 0.25\sqrt{\chi_2} - 0.2 \geq 0 \quad (9b)$$

Letting  $\lambda$  and  $\mu$  denote the Lagrangian multipliers, the Lagrangian function becomes:

$$\begin{aligned} L &= f(\chi_1, \chi_2) + \lambda g_1(\chi_1, \chi_2) + \mu g_2(\chi_1, \chi_2) \\ &= 0.75(1 - \chi_1) - 0.25\chi_2 + \lambda(0.75\sqrt{\chi_1} + 0.25\sqrt{\chi_2} - 0.25) + \mu(0.25\sqrt{\chi_1} - 0.25\sqrt{\chi_2} - \\ &0.2) \end{aligned} \quad (10)$$

To find the point where  $f(\chi_1, \chi_2)$  is maximum, we need to find the partial derivatives of L and set them equal to 0 ( $\partial L/\partial \chi_1 = 0$ ,  $\partial L/\partial \chi_2 = 0$ ,  $\partial L/\partial \lambda = 0$ ,  $\partial L/\partial \mu = 0$ ):

$$\partial L/\partial \chi_1 = -0.75 + 0.375\lambda(1/\sqrt{\chi_1}) + 0.125\mu(1/\sqrt{\chi_1}) = 0 \quad (11)$$

Multiplying (11) by 8 and moving the constant to the right hand side gives us:

$$\partial L/\partial \chi_1 = (1/\sqrt{\chi_1})(3\lambda + \mu) = 6 \quad (12)$$

Now, for the second partial derivative ( $\partial L/\partial \chi_2$ ):

$$\partial L/\partial \chi_2 = -0.25 + 0.125\lambda(1/\sqrt{\chi_2}) - 0.125\mu(1/\sqrt{\chi_2}) = 0 \quad (13)$$

Multiplying (13) by 8 and moving the constant to the right hand side gives us:

$$\partial L/\partial \chi_2 = (1/\sqrt{\chi_2})(\lambda - \mu) = 2 \quad (14)$$

The partial derivatives with  $\lambda$  and  $\mu$  are:

$$\partial L/\partial \lambda = 0.75\sqrt{\chi_1} + 0.25\sqrt{\chi_2} - 0.25 = 0 \quad (15)$$

$$\partial L/\partial \mu = 0.25\sqrt{\chi_1} - 0.25\sqrt{\chi_2} - 0.2 = 0 \quad (16)$$

Adding (15) and (16) gives us the solution to  $\sqrt{\chi_1}$ :

$$\sqrt{\chi_1} = 0.45 \Leftrightarrow \chi_1 = 0.2025 \Leftrightarrow I(1) = 0.2025 \quad (17)$$

Substituting this value into either equation (15) or (16) gives us

$$\sqrt{\chi_2} = -0.35 \Leftrightarrow \chi_2 = 0.1225 \Leftrightarrow I(0) = 0.1225 \quad (18)$$

If we substitute the values in (17) and (18) into (12) and (14), we can obtain the values for the Lagrange multipliers  $\lambda$  and  $\mu$ . The results yield  $\lambda = 0.5$ ,  $\mu = 1.2$ .

The expected utility of the principal is now:

$$\begin{aligned} E(U_P) &= E\{x - I(x)\} = (1 - I(1))*0.75 + (0 - I(0))*0.25 \\ &= (1 - 0.2025)*0.75 + (-0.1225)*0.25 = 0.5675 \end{aligned}$$

The expected utility of the agent when he provides a high level of effort is:

$$\begin{aligned} E(U_A(e=1)) &= E\{\sqrt{I(x)} - 0.2e\} = (\sqrt{I(1)} - 0.2)*0.75 + (\sqrt{I(0)} - 0.2)*0.25 \\ &= (0.45 - 0.2)*0.75 + (-0.35 - 0.2)*0.25 = 0.05 \end{aligned}$$

$$\begin{aligned} E(U_A(e=0)) &= E\{\sqrt{I(x)} - 0.2e\} = \sqrt{I(1)} * 0.5 + \sqrt{I(0)} * 0.5 \\ &= (0.45)*0.75 + (-0.35)*0.5 = 0.05 \end{aligned}$$